Designing for Children
- With focus on 'Play + Learn'

Design criteria for designing of spatial Number Sense Manipulatives

Ekta, Surender, Indian Institute of Technology Kanpur, sekta@iitk.ac.in
Koumudi, Patil, Indian Institute of Technology Kanpur, kppatil@iitk.ac.in

Abstract: Since the 19th century, there has been a shift from a lecture and recitation-based education towards activity-based learning through the involvement of the learner's senses. The educational manipulatives have played a vital role in this transition by enabling the construction of knowledge through 'concrete operations', before moving on to 'formal operations'. The design process of these manipulatives has seen the participation of child-users at different levels; from the earliest role of children as the 'user', to 'tester' to a recent, more inclusive role as the 'informant'. This study explores the role of child-informants in the design process of designing spatial based number sense manipulatives, that spatialize both the numeral and quantity aspects of a number, and thus derive the design criteria for designing such manipulatives. A lack of similar existing manipulatives coerced the designer to follow an exploratory and iterative research approach, which enabled multiple cycles of quick prototyping and testing, after regular inputs from the child-informants. Thus, a systematic, iterative prototyping process was followed, which was documented, analyzed, and reflected upon through the action research method. Consistent feedback from the child-informants in the iterative action cycles directly impacted the design of subsequent prototypes and resulted in the derivation of design criteria for designing the final set of manipulatives.

Key words: Manipulatives, child-informants, Number Sense, Spatial Sense, Prototyping

1. Introduction

Since the 19th century, there has been a shift from a lecture and recitation-based education towards activity-based learning through the involvement of the learner's senses. This involved contributions from Heinrich Pestalozzi (1803), who argued for 'things before words, concrete before abstract', Friedrich Froebel and Maria Montessori (1912) who build manipulative materials for kindergarteners focusing on 'education of their senses' (Resnik, 1998), and Jean Piaget (1972), who theorized that children must first construct knowledge through 'concrete operations', as acquired with the manipulatives, before moving on to 'formal operations'. Recently, Seymour Papert has further stressed upon concrete thinking stating that it is no less important than abstract knowledge, which is detached from the knower (Ackermann, 2001).

Today, several studies elicit the importance of manipulative materials, and they are well-established in the classrooms, especially in the early grades (Sarama & Clements, 2016), (Resnick et al., 1998), (Boggan, Harper & Whitmire, 2010). The design process of these manipulatives and other child-centric learning products has seen the participation of child-users at different levels. Some of the earliest studies on designing of technology products for children involved children as the 'user'. With this role, the child is considered a user of the product while the adult looks to understand the child's activities with various methods (Druin, 2002).

In the late '90s, children's involvement in the design process grew further as they were put in the role of 'testers'. For example, children as 'testers' were allowed to construct their own paths to knowledge (an approach termed as constructionism), during the development of the programming language 'Logo', headed by Professor Seymour Papert. With this role, children test prototypes of emerging technologies, with a goal of shaping the new technologies before the release of these as commercial products into the world (Druin, 2002).

More recently, children have been brought in as 'informants' into the design process. Similar to the role of testing, this involves observing children interact with the existing technologies, which provides new design directions to the design team. These directions, if not expressed directly by the children, may be implied by their actions (Druin, 2002). However, unlike the role of testers, informants are brought into the design process more often depending on the requirements and thus have a direct impact on the design of a product (Druin, 2002).

The role of children as informants is particularly useful for a designer in a scenario of designing a novel product, where there is limited or no information to begin the design process. It also suits the requirement of gathering feedback on products from very young and verbally inarticulate children, who tend to display their responses by 'acting out' instead of discussing like adults (Druin, 2002).

Thus, the study aims to explore the role of child-informants in the design process, by considering them an entirely different user population than adults, with their own culture, norms, and complexities (Druin, 2002). This exploration is done under the context of designing spatial based number sense manipulatives for the young cohort.

1.1. Spatial Sense and Number Sense-

Spatial Sense, primarily comprising of Spatial Visualization (SV from hereon), is the ability of building and manipulating mental representations of two-and three-dimensional objects and perceiving an object from different perspectives (NCTM, 2000, p. 41). Mental Rotation is a key aspect of SV, which is defined as the ability to visualize how an object might look like when rotated in 2D or 3D space (NCTM, 2000).

Number Sense (NS, hereon) is interpreted in several ways in the literature (Berch, 2005). However, this study sees it as an ability to be fluent with numbers as defined by Fosnot and Dolk (2001). That is, knowing how a number can be composed and decomposed and using that information to be flexible and efficient with solving problems (Boaler, 2015). For example, knowing 25 is composed of 20 and 5 or two 10s and a 5.

While several studies have established a strong correlation between a child's spatial visualization skills (MR in particular), and Number Sense, (Laski, Casey, Yu, Dulaney, Heyman, & Eric Dearing, 2013), (Casey, Lombardi, Pollock, Fineman, & Pezaris, 2017), there are few manipulatives which explicitly integrate these two aspects (Ontario Ministry of Education, 2014), (Davis, 2015). And fewer still, spatialize the numbers entirely in terms of both quantity and numeral or the 'visual number form' aspects (Kadosh & Dowker, 2015). Thus, the study aimed at designing an entirely spatial manipulative for imparting the concept of NS by involving the child-informants in the design process and thereby deriving the design guidelines or criteria for creating such manipulatives.

While an in-depth study on the prototyping process for designing these manipulatives was presented in the DRS Learn XD 2019 conference at Ankara, Turkey (paper publication under process), this paper enumerates the role of child informants in shaping the design process thus leading to the emergence of plausible design guidelines for such manipulatives. These emergent guidelines are only a part of a larger ongoing academic research work.

2. The Research Method

Our research objective required the creation of a novel manipulative instead of testing an existing one, owing to sparse initial reference. Therefore, designer's limited initial understanding of designing an all-new manipulative demanded an exploratory and iterative research approach, which enabled multiple cycles of quick prototyping and testing, after regular inputs from the child-informants. The child-informants being verbally inarticulate, also necessitated the collection of their feedback through observation of their actions instead of direct questioning. These study requirements seemed possible to be accomplished through the action research method (Kemmis & McTaggart, 2005). Thus, a systematic, iterative prototyping process was followed, which was documented, analyzed, and reflected upon through the action research method.

The study was conducted with eight preschool students aged between (4-4.5-year-old) at the Redbird Kindergarten School, Nankari village, Kanpur, Uttar Pradesh in India. The students here hailed from a lower socio-economic stratum with little or no possibility of gaining knowledge through additional tuitions. As described by the school Principal and teachers, the students had recently learned numbers up till forty, by mostly counting and numeral identification. However, the concept of quantity was still not clear, and number composition and decomposition were completely unfamiliar. This proved to be a fertile ground to introduce numbers as spatial entities to the students before they were trained in a different worldview of numbers (Surender, 2019).

Requisite permissions from the Principal as well as the parents were taken in a parent-teacher meeting. To maintain the privacy of the subjects, their names have been altered.

3. The Design Process

Due to the lack of availability of similar manipulatives, it was not possible to involve the child-informants at the very start of the design process by observing them while using these manipulatives (Druin, 2002). Thus, the designer found it appropriate to design an initial low fidelity prototype based on the current understanding of subject matter such that it fulfilled the necessary design requirements. This prototype was then used as a starting probe (Sanders & Stappers, 2014) against which the child-informants provided feedback through their actions. These feedbacks informed the designer of further modifications in the prototype, which was again tested with the children. These iterative action cycles of prototyping and testing were subsequently followed which are explicated below. The cycles 1 to 4 deal with designer's hands-on explorations based on initial understanding of the subject matter to arrive at the first low fidelity prototype-probe. The subsequent cycles thereon involve the participation of the child-informants in the design process.

3.1 Action Cycle 1

3.1.1 Plan

The design process commenced with the objective of designing a beginner prototype-probe that could spatialize both the 'quantity' as well as the 'numeral' aspects of numbers, and thereby be able to impart learning of number composition and decomposition through SV, i.e., Mental Rotation. As the study progressed with designing of more prototypes followed by subsequent testing of these with the child-informants, new design criteria emerged along the process.

In the absence of a definite direction toward the problem solution, the designer began prototyping with the simple idea of correlating the two main variables of the design problem- numbers and spatial comprehension. This was done by representing each number by a basic geometric shape, say a square. That is a 1 can be equivalent to a single square; consequently, a 2, which is nothing but 1+1, can then be composed of two squares.
In this concept, the quantity of a number is equivalent to the number of squares, and the numeral is nothing but the rectangular shape formed by the unique composition of these squares. The concept of number composition and decomposition was then possible to be shown by representing each number rectangle in terms of various smaller number rectangles (Surender, 2019).

3.1.2 Action

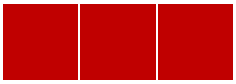
The author created colorful Number-strips out of paper to externalize the above plan. The strips were colour-coded for each number as shown below-

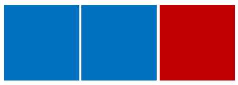
1= a single square of red colour 

2= a blue rectangle formed by two squares 

3= a yellow rectangle formed by three squares 

Therefore, the quantity of number 3 was equivalent to three yellow squares in the spatial domain. More importantly, 3 could be composed of in terms of various other smaller Number-strip combinations. For instance,

3 = 1+1+1 is equivalent to 3 single squares 

3=2+1 is equivalent to two blue squares and a single red square 

'3' can also be represented as a single yellow rectangle to represent three as a whole

This way, the authors were able to compose various numbers with combinations of other Number-strips, as shown in Figure 1.

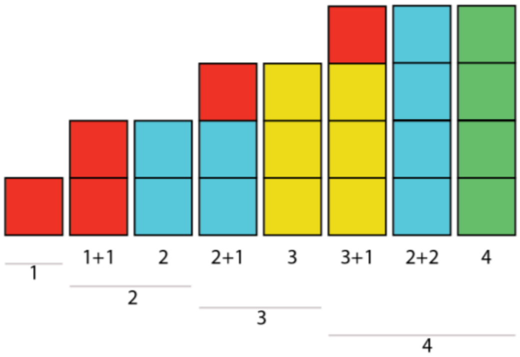


Figure.1 The Number-strip prototype. Source: (Surender, 2019)

3.1.3 Reflect

The designer, i.e., the author created Number-strips for numbers 1 to 15 and tried to emulate the user scenario by composing and decomposing various numbers with these strips. In doing so, the author faced difficulty while identifying each Number-strip with its corresponding number. This was because the rectangular shape, which represented the 'numeral' aspect of a number (in the spatial domain), was constant for all the numbers; that is, a new number was nothing but another rectangle only with an increased length, instead of a unique rememberable shape. Also, visually distinguishing the rectangular numbers by an only change in their lengths was very challenging. This increased the dependency on colour for identifying each Number-strip, which was again tedious due to a large number of colours and numbers. The designer's exploration gave rise to the design criteria that *the prototype should involve unique shapes instead of constant ones. It should also be least dependent on colours and on the change of lengths of the shape for ease of identifying and recalling of the number.*

Since the strip prototype was unsuccessful, the designer explored more ideas which correlated the 'numeral' aspect with 'unique shape' instead of 'colour', in addition to spatializing the quantity aspect. This shift on focus to the shapes led the designer to explore the realm of patterns in mathematics, which in turn led to the concept of fractals.

3.2 Action cycle 2

The exploration of the Fractal concept resulted in the creation of Number-Shapes based on the concept of the tree fractals, as shown in Figure 2 (Surender, 2019).

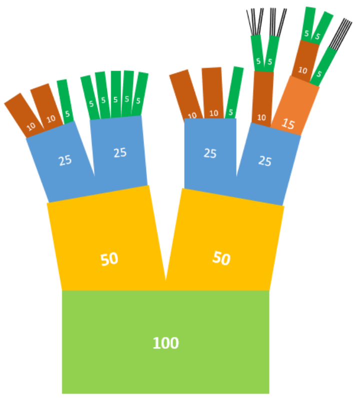


Figure.2 The Tree fractal prototype- decomposition of the number ‘100’. Source: (Surender, 2019)

However, this concept also failed as the same problem of unique shape resurfaced among the closely spaced numbers. For example, it was difficult to visually distinguish between the two configurations of 8 = 5+3 and 8= 4+4, as shown in Figure 3. This again increased the dependency on colour to identify a number. The prototype also did not facilitate mental rotation (Surender, 2019).

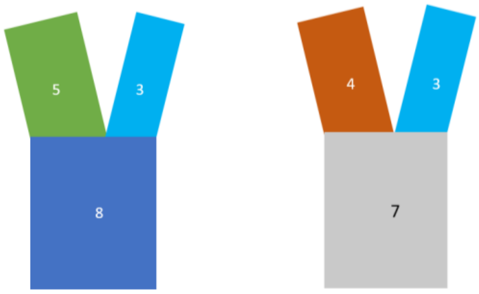


Figure.3 Tree fractal representation for 5+3=8 and 4+3=7. Source: (Surender, 2019)

Thus, the designer now searched for new platforms which would enable the creation of unique shapes or patterns.

3.3 Action Cycle 3

3.3.1 Plan

The search for unique shapes led the designer to the discovery of Islamic tessellations, which involved the creation of several unique 2D patterns on the same grid. Islamic tessellations are similar to fractals with infinitely repeating patterns. They are geometric designs which are created by fitting polygonal shapes together, without any gaps. These polygons are drawn on either a square or an equilateral triangle-based grid, as shown in Figure 4 (IAGD, 2004).

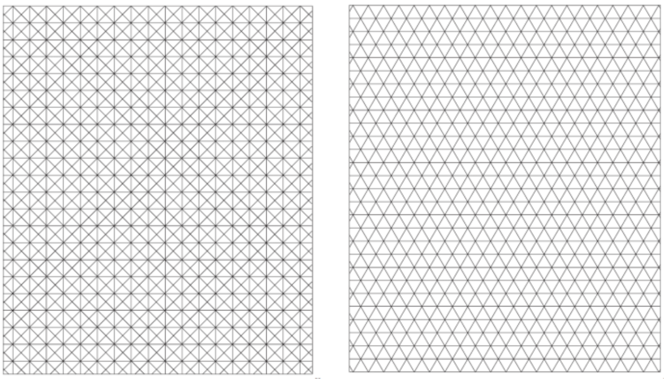


Figure.4 The square and equilateral triangle-based grid. Source: (IAGD, 2004)

The tessellation grids provide ample affordance for construction of symmetric patterns, a trait which was most welcome in this study because symmetry-based training has proven to positively affect student's spatial visualization ability (Nes, 2009), (Davis, 2015, p. 92), (Surender, 2019).

Also, the same tessellation enabled viewing of multiple patterns in it, which made it further favorable for the study. For example, the tessellation in Figure 5, can be seen as a pattern of repeated flowers as well as that of repeating diamonds. This can be considered similar to the viewing of the same number in terms of various smaller number compositions.

Seeming well-suited to our study objectives, this concept was further explored in detail.

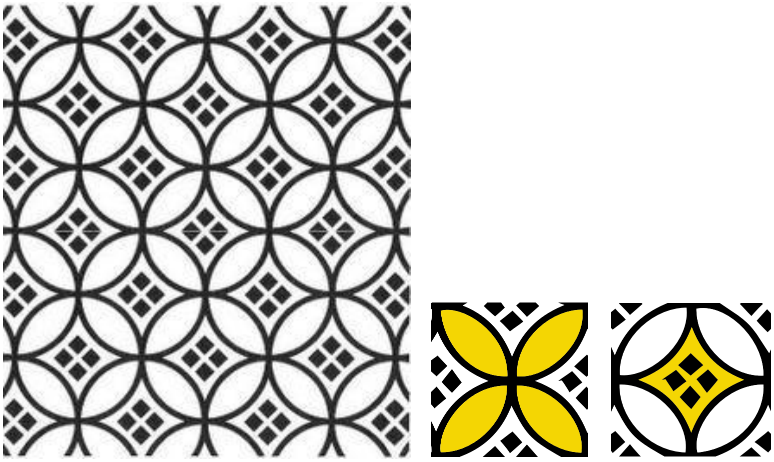


Figure.5 Multiple patterns in a tessellation. Source: (Surender, 2019)

3.3.2 Action

A square grid (Figure 4) was selected for exploring the construction of symmetric shapes. A '1' was represented by any single right-angled triangle in the grid, as shown in Figure 6. The subsequent numbers were then formed inside the single grid element (Figure 6) with the following constraints (Surender, 2019):

- A number x can be formed with x number of right-angled triangles, thus spatializing the quantity of a number
- These triangles must be arranged in the grid such that the sides or the vertex of any two triangles must be joined so as to form a symmetrical pattern, thus spatializing the visual number form of the number

Based on the above self-imposed design constraints, the designer created all the possible tessellations (Number-Shapes from hereon), from 1 to 10. These Number-Shapes for numbers 1-5 are shown in Figure 6.

Thus, a number is spatialized as follows- the quantity aspect of a number, say, 3 is equivalent to three connected right-angled triangles in the spatial domain, and the numeral aspect of 3 is equivalent to the unique symmetric patterns created by the triangles as shown in Figure 6 (Surender, 2019).

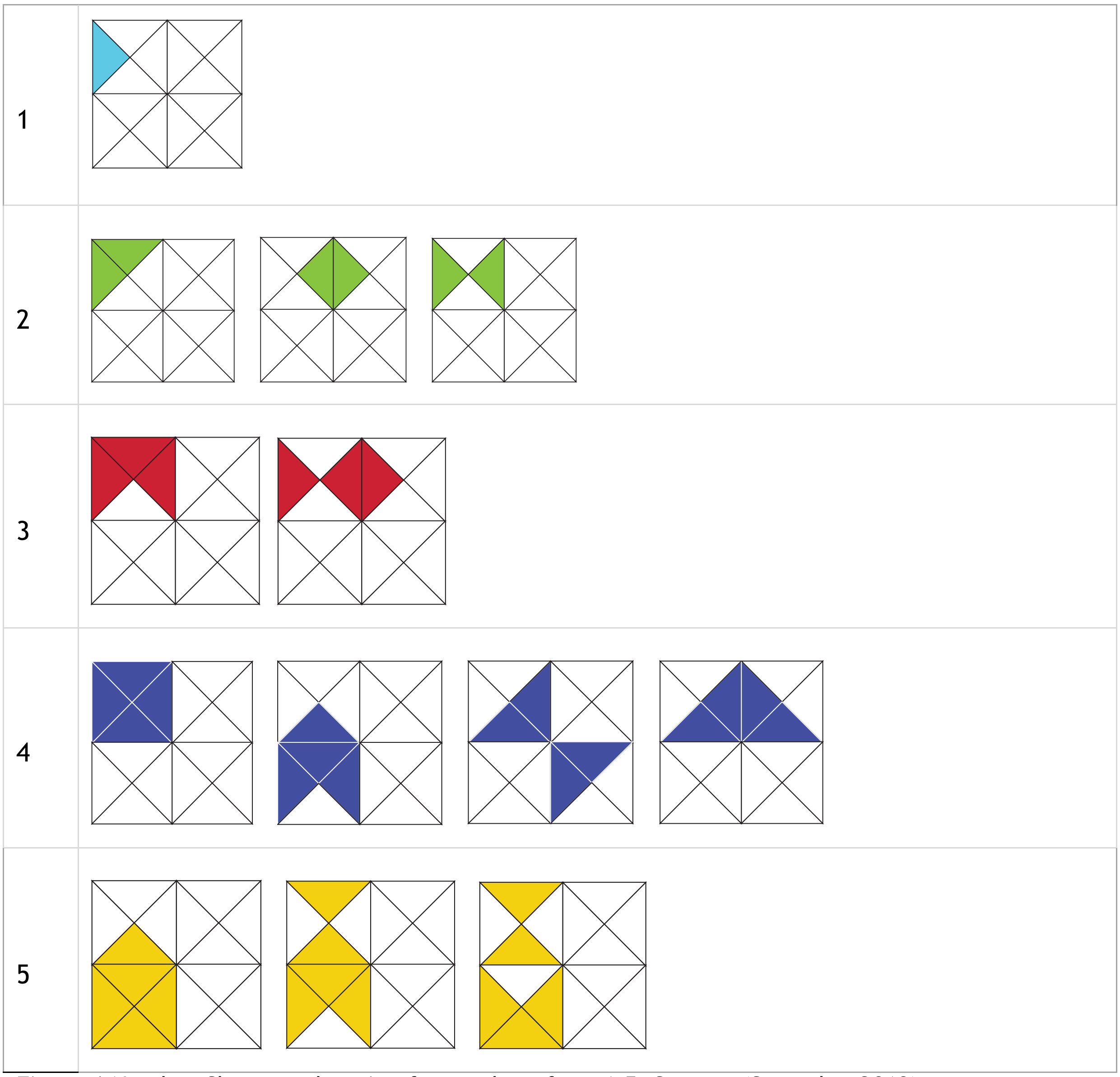


Figure.6 Number-Shape exploration for numbers from 1-5. Source: (Surender, 2019)

Several Number-Shapes were then composed and decomposed by the designer. For example, Number-Shape for the number ‘4’ (Figure 7(a)) can be constructed in terms of various number compositions- four 1s, a 3 and a 1, and two 2s (Figure 7(b), 7(c), 7(d), respectively). It should also be noted that the Number-Shape of ‘two’ (the green square) in the ‘4’ is used in a rotated orientation, thus indicating the inherent ability of this concept for aiding mental rotation (Figure 7) (Surender, 2019).

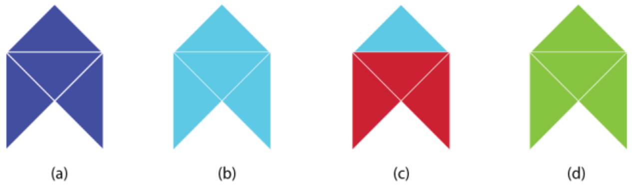


Figure.7 Various Number-Shapes of the number ‘4’. Source: (Surender, 2019)

3.3.3 Reflect

Thus, symmetric tessellations not only proved effective in spatializing numbers for their ‘quantity’ aspect but also their ‘numeral’ aspect by allowing the creation of a unique set of patterns for each number. This made identifying and remembering of the numbers in the spatial domain possible after a little familiarization.

The symmetry offered by the tessellation concept also led to the introduction of a new design criterion in the study that *the manipulative must provide affordance to enable the construction of symmetric geometric shapes*.

The tessellation concept seamlessly enabled the composition and decomposition of Number-Shapes in terms of various smaller Number-Shapes (Figure 7). In addition to this, the concept also intrinsically incorporated mental rotation.

3.4 Action-Cycle 4

3.4.1 Plan

Since the tessellation concept seemed to fulfill the study objectives holistically, it was manifested into a tangible physical manipulative, to act as the beginner probe for testing it with the child-informants.

3.4.2 Action

The designer cut out several right-angled triangles out of cardboard and tinkered with different connections between them to enable the construction of various Number-Shapes. Amongst all the alternatives, a simple thread-based link was found to sufficiently fulfill the concept requirement. It was also inexpensive and quick for prototyping and testing.

These links were executed as follows- the vertex of each triangle was connected to the adjacent ones with a thread. These Threaded-Triangles or TTs were made for each number from 1 to 10, such that the TT for a number x comprised of x number of triangles, and enabled formation of various Number-Shapes for the number x. These Number-Shapes were the same set of unique triangular, symmetric patterns which were developed in Action-Cycle 3. For instance, the TT for number 4 and its various Number-Shapes are shown in Figure 8 (Surender, 2019).

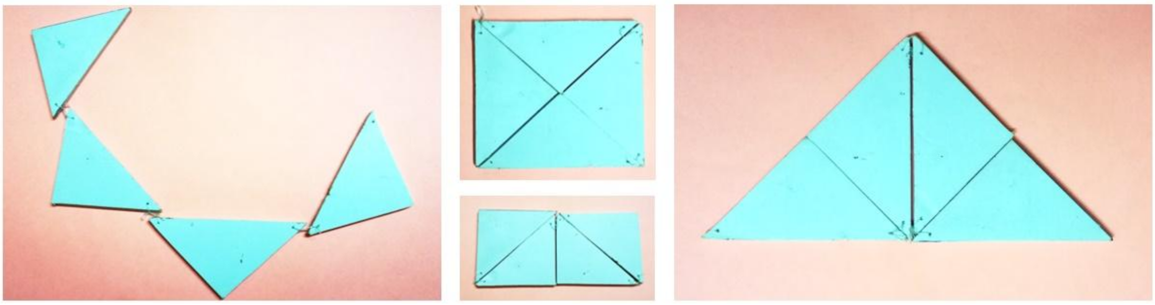


Figure.8 The Threaded-Triangle prototype for number '4'. Source: (Surender, 2019)

3.4.3 Reflect

Designer's tinkering with the threaded links divulged another design criterion, which was not realized earlier- *The connections in the prototype must offer sufficient freedom of movement of triangles for the ease of exploration while construction of Number-Shapes, as well as for ease of iterations during error corrections.*

The Threaded-Triangles reaffirmed previous cycle's criteria of ease of identifying, remembering, and recalling of Number-Shapes, though the initial learning curve could differ between children. It also offered several possibilities of physically manipulating and constructing various Number-Shapes, by flipping, twisting, and moving the triangles. Hence the beginner probe was now in a position to be tested the TTs with children to validate the concept (Surender, 2019).

3.5 Action cycle 5

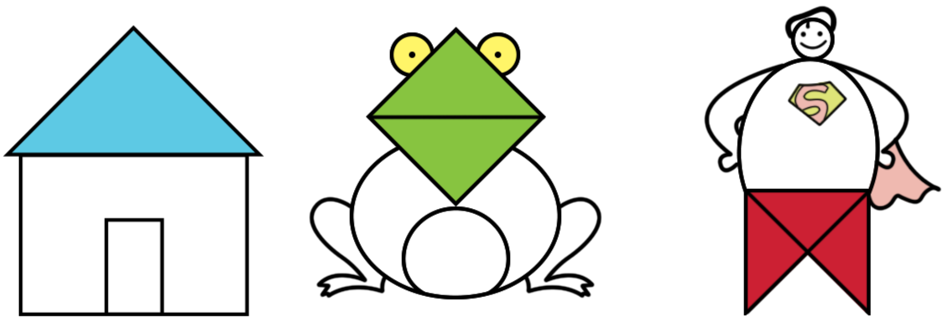
3.5.1 Plan-

Since the concept of Number-Shapes and Threaded-Triangles was entirely new to the children, the designer planned to deploy the TT prototype in a phase by phase manner. Phase-1 would involve introducing children to the concept of triangular Number-Shapes, followed by providing them the unthreaded independent triangles to create the Number-Shapes on their own. This was to be followed by Phase-2, involving composing and decomposing of various Number-Shapes with the TT prototype by the children (Surender, 2019).

3.5.2 Action

After obtaining the requisite permissions, the author carried out a small workshop in a class of eight students at the Redbird School. The author demonstrated the concept of Number-Shapes by verbally explaining as well as constructing the Number-Shapes with some free triangles on the blackboard. However, this method of instruction turned out to be ineffective since children displayed poor attention span. They were also too young to patiently sit still and carefully listen to the verbal instructions. On discussing the scenario with the Principal, it came to be known that while lack of attention was typical behavior of the children, they did display acute listening tendency during his daily storytelling sessions. Children also enjoyed these sessions.

Thus, the Number-Shape concept was now planned to be administered with the help of contexts and stories. For this, the triangular Number-Shapes were put in a picture-based context (Figure 9), followed by a story based on these shapes. These contexts were as follows- The number ‘1’, which was a single right-angled triangle, shown as a roof of the hut, the number ‘2’ as a frog face and ‘3’, as the superman’s pant, and so on (Figure 9 (a),(b),(c), respectively).



(a) (b) (c)

Figure.9 Number-Shapes put in picture-based contexts

After familiarizing children with the Number-Shapes, integration of these shapes with actual numbers was done through a fictitious story of a young boy, named Kaku, and his world where the numbers were represented as symmetric triangular patterns (i.e., the Number-Shapes). The story was verbally narrated using pictorial aids, and the concept of Number-Shape formation using triangles was dispensed through Kaku’s father tutoring the same in the story. This was followed by providing each child with loose triangles to let them explore the creation of various Number-Shapes to, in turn, help Kaku learn.

While children were able to reconstruct the demonstrated symmetric patterns, they also showed a tendency to create new symmetric and non-symmetric shapes of their own. Many of them were also excited after creating a pattern and tried to show it to their friends by either inviting them or by taking their creations to friend's seat. Children often moved around the class, sometimes with the triangles, and sometimes without it.

3.5.3 Reflect

The cycle revealed that the four-year-old informants displayed little patience and a sparse attention span, particularly towards verbal instructions. Thus, informing the designer with the criterion that *Instructions and activities for a young cohort as this, should be brief, understandable, and interesting, in order to be effective*.

The child informants responded positively towards the context and storytelling-based concepts. They not only paid attention to but also understood the author's instructions as they actively created several demonstrated as well as self-thought patterns from the loose triangles. Many were also able to recall the Number-Shapes along with their contexts during the interim evaluations. During the following lunch session, one of the students, interestingly, also reckoned the presence of a triangle in his lunch box on seeing a chapati folded nearly in the form of a triangle.

These actions of the child informants suggested another design criterion that *Visual context and storytelling are effective means of dispensing unfamiliar instructions and hence can be incorporated in the manipulative*.

In Phase 2, children showed the tendency to create asymmetric shapes with the given triangles, in addition to the symmetric ones. Similarly, the TT prototype also provided the affordance for construction of both asymmetric and symmetric Number-Shapes. However, restricting children to only symmetric shapes was desirable according to the study objectives. The child-informants also revealed their expectation of portability from the tool through their actions of moving around with the triangles. These insights called for a redesign of the TT prototype so as to restrict children to symmetric shape formations and also for making it portable.

3.6 Action cycle 6

3.6.1 Plan

The course of the design process, which earlier dealt with testing of the TT prototype, was redirected after the insights provided by the child-informants in the previous action cycle. The TT prototype was now modified to accommodate the following features -first, constraining the user to build only symmetric Number-Shapes, and second, making it portable.

3.6.2 Action (by the Designer)

Explorations done based on the required features resulted in the creation of a Cut-out tool. It consisted of the following parts- (1) a base with a Cut-out niche that constrained the shapes to only symmetric ones and (2) a set of free Number-Shape cards that fit into the Cut-out niches. These are shown in Figure 10 (Surender, 2019).

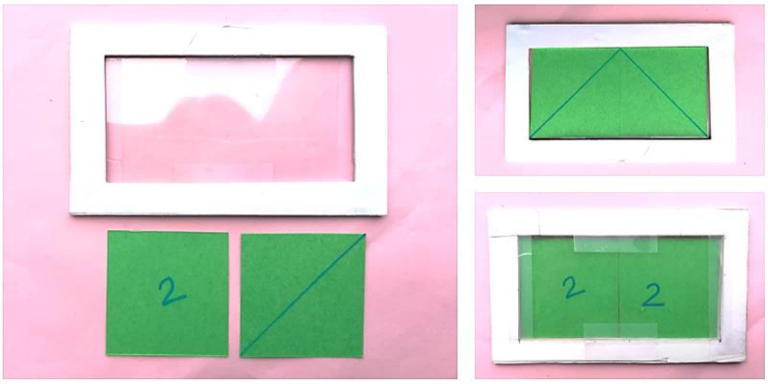


Figure.10 The Cut-out prototype for ‘4’ with two ‘2s’. Source: (Surender, 2019)

The Number-Shape cards contained the line-diagram of the Number-Shape pattern on the front side and the respective numeral written at the backside (Figure 10). A transparent sheet was pasted at the back of the Cut-out base to hold the placed Number-Shape cards and to enable viewing of the numerals, written at the back. It also ensured portability. Such Cut-outs were created for numbers from one to five, some of which are shown in Figure 11 (Surender, 2019).

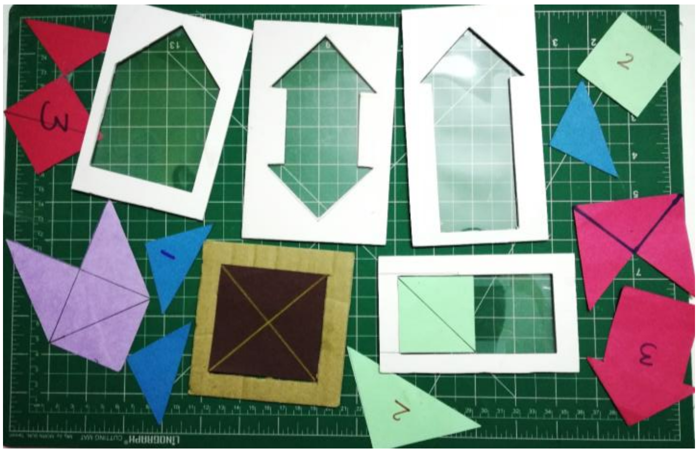


Figure.11 Cut-outs for numbers- ‘4’ and ‘5’ with paper Number-Shapes. Source: (Surender, 2019)

The Cut-out prototype facilitated construction of a Number-Shape through the placement of various combinations of smaller Number-Shape cards in the Cut-Out base.

This way, the process of construction and deconstruction of Number-Shapes was also quick and less tedious, as compared to the TT tool. The prototype also enabled rotation of various smaller Number-Shape cards to be placed into the base, thus addressing the mental rotation requirement of the study.

This revised prototype was again deployed with the child-informants for validation and feedback.

3.6.3 Action (by the children)

The author introduced children to the Cut-out manipulatives for numbers 1, 2, 3, 4, and 5 and demonstrated the filling up of Cut-out bases with various smaller number compositions. Children were then given a Cut-Out each to explore the prototypes on their own (Surender, 2019).

Children learned to construct various Number-Shapes with the Cut-out manipulatives. After filling up the niches with the Number-Shape cards, they also counted the number of triangles to state which number had been created.

3.6.4 Reflect

As planned, the prototype was successful in restricting the students to create only symmetric Number-Shapes. It also enabled them to explore the composition and decomposition of the same Number-Shape with multiple smaller Number-Shapes, while also providing the affordance for quick iterations and error corrections. However, the prototype was unable to make children speak of number construction in terms of smaller numbers as a whole, as they were counted only as ones. For example, after the construction of a four with two 2s (Figure 10), children counted the four individual triangles of the Number-Shape 4 instead of viewing it as a composition of two 2s. This way, the actions of child-informants informed the designer of a new design criterion that is, *the manipulative must provide affordances of viewing the composed Number-Shapes in terms of various smaller Number-Shapes instead of only as ones.*

3.7 Action Cycle 7

3.7.1 Plan

The problem of viewing number composition in terms of numbers other than 1’s was attempted to be tackled using the previously successful storytelling approach.

3.7.2 Action

The story was now tailored to emphasize Kaku's viewing of the same Number-Shapes in terms of different smaller number compositions. The verbal narration of the author was again complemented with the scenario sketches, as shown in Figure 12.

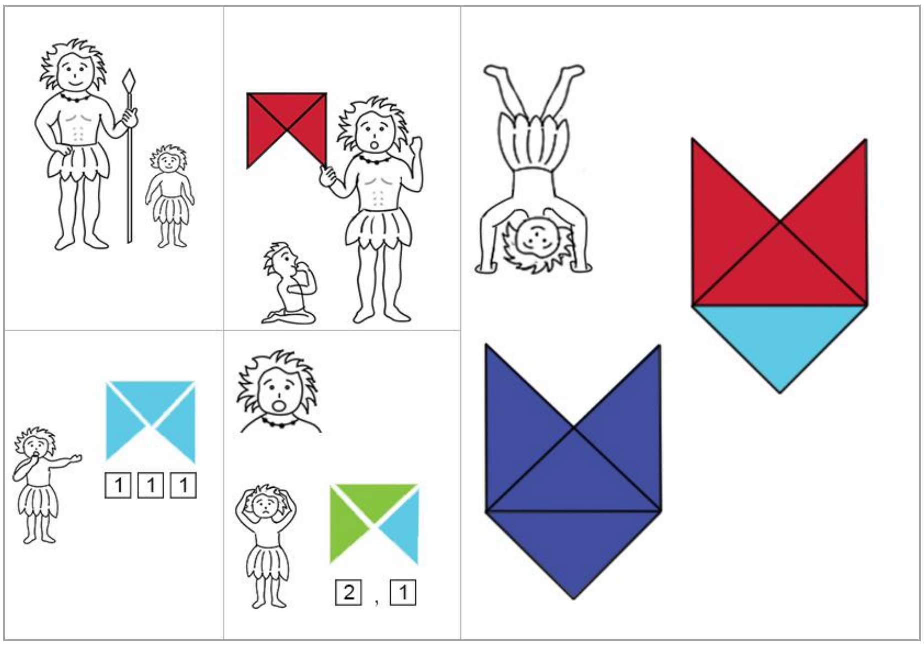


Figure.12 Scenario sketches during the narration of Kaku's story

This storytelling session once again turned out to be successful as children paid greater attention, and better understood the concept of number composition. It was interesting to note that specific real-world components of the story, such as Kaku's standing upside down (Figure 12), immensely interested the children.

As an effect, few children began seeing numbers in terms of other smaller numbers. For example, one of them said that a '2 and a 1 make a 3', and another said that 'a 3 and a 1 make a 4'. However, a majority of the children still spoke of number composition in terms of 1s.

3.7.3 Reflect

Action cycle 7 reinforced the effectiveness of the storytelling criterion. The affinity of children towards real-world components in the story led to a new criterion that – *relatable real-world elements can be incorporated into the stories for raised interest and better engagement of young children.*

It was also observed that all children had now started remembering and using the Number-Shapes by their context names. For example, during their explorations, one of the students explained to the other- "To make that 4, take a superman's pant and add a 1 to it" indicating that context, not only acted as a means to remember the shape but also as a medium for communication.

The mere physicality of manipulatives does not guarantee the learning of the mathematical idea they carry. Students may benefit from manipulatives to build meaning initially, but they must reflect on and talk about their actions with manipulatives to do so (Sarama & Clements, 2016). Providing a context to the Number-Shapes in the form of a super man's pant or a frog face, gave students a medium to reflect and talk about their actions. Thus, leading to the design criterion that- *Putting the abstract Number-Shapes in a context is important, as it enhances children's attention and also acts as a medium for communication among the child-informants.*

Though children were now able to begin to speak in terms of other smaller numbers, many still resorted to counting the individual triangles instead of considering the smaller Number-Shapes as a whole; due to which questions like 'how many 'twos' make a 'four'?', still remained unanswered by all. Therefore, the problem of designing a suitable prototype for enabling viewing of composing and decomposing of numbers in terms of numbers other than ones still persisted.

3.8 Action cycle 8

3.8.1 Plan

On reflecting upon the workshop experience so far, it was realized that the Cut-out prototype had emphasized more on the shape aspect of the Number-Shapes than their quantity aspect. For example, the child had to focus more on the outline of the Number-Shape to decide if it can fit into the Cut-out base and not so much on the number of such shapes being selected to fill up the base. Thus, failing to emphasize the quantity.

Shifting the child's focus from the 'shape' to the 'quantity' aspect of the Number-Shapes forces her to see the Number-Shapes as a single entity. That is, in order to make a student say that the Number-Shape of the number 3 is made up of – one Number-Shape of 2 and one Number-Shape of 1, they have to be first shown multiple units of Number-Shapes of 2 and 1 out of which a single unit of each has to be selected (Figure 13). Thereby forcing the students to see the entire Number-Shape of 2 as a single unit and not as two ones. Hence, designer's objective now shifted towards enhancing the prototype's emphasis on the quantity aspect of the Number-Shape, such that it may trigger the student to say "I will take one 2 from here, and one 1 from here to make a 3" (Surender, 2019).

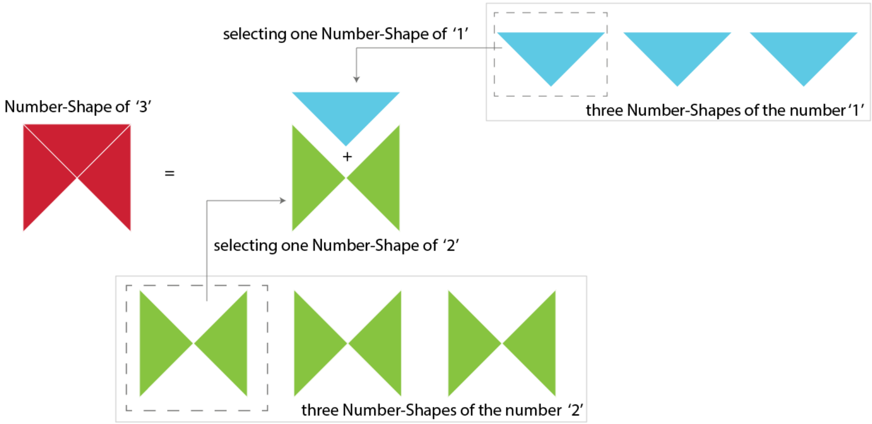


Figure.13 Composition of number three through the disc tool concept. Source: (Surender, 2019)

3.8.2 Action

Brainstorming and ideation on this plan led to the creation of a disc prototype. It comprised of a central cardboard with the Number-Shape Cut-out at its centre and two spinning discs attached at its corners (Figure 14). Each disc consisted of multiple units of smaller constituent Number-Shapes. The central Cut-out number was created by spinning the corner discs and selecting any one of these multiple smaller Number-Shapes from each disc to fill the central Cut-out. For example, a tool for number 4 contained two discs, disc-1 having three 3s and disc-2 with four 1s (Figure 14). The central Cut-out cavity, representing the number 4 was filled by spinning the discs and choosing one 3 from disc-1 and one 1 from disc-2. Thus, emphasizing on the concept of viewing smaller numbers as a whole by selecting one of it, at a time. Similar tools were designed for numbers 2, 3, and 4 and presented to the students (Surender, 2019).

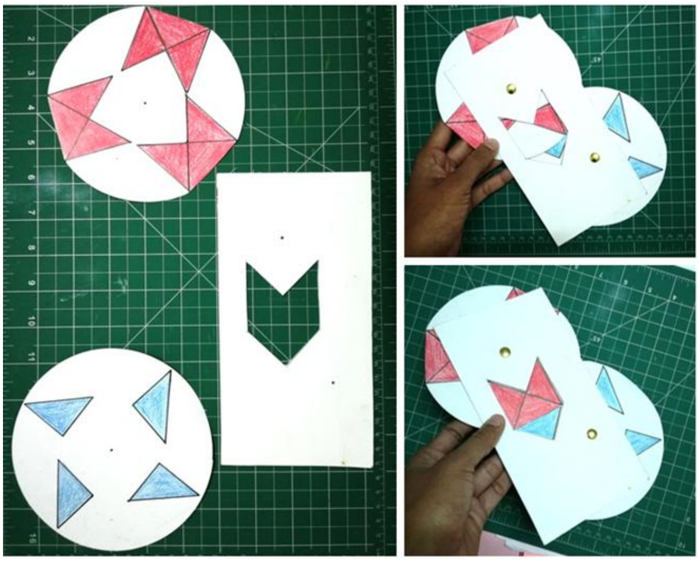


Figure.14 The disc tool for number 4 with composition 3+1. Source: (Surender, 2019)

3.8.3 Reflect

All the children were now able to see the constituent Number-Shapes in terms of various other smaller Number-Shapes, instead of just ones. They began to speak of number composition in terms of various smaller numbers such as "2 and 1 combined to make a 3" and "two 2s make a 4", etc (Surender, 2019).

While playing with the disc prototype of the number 4, the discs accidentally got dismantled, when a child fixed it, and excitingly showed it to his friend "See Shyam, 3 and 1 combine to form a 4". This indicated that students were more engaged while 'making' the manipulative, rather than when solving a problem with it. This action offered another design criterion that- *For better engagement, the manipulative should provide more opportunities for children to 'make' it on their own.* This finding also aligns with Papert's constructionism that suggests that learners actively learn through 'making' external artefacts as they engage in a conversation with their own or other people's artefacts; these conversations boost self-directed learning, and ultimately facilitate the construction of new knowledge (Brennan, 2014), (Ackermann, 2001).

Toward the end of the session, children spun the discs like a fan or often simply ran the prototype on a bench like a car, which was an entirely unforeseen application of the tool. This action of the child-informants suggested another interesting design criterion *that it is essential to allow children free time to play with the manipulatives for internalizing the learnings and to facilitate ludic play and exploration*, as also mentioned by Boggan et al. (2010). These playful explorations by children can reveal some interesting, unprecedented applications of the manipulative to the designer.

The design criteria developed so-far would further guide the designer for the design of a final spatial Number Sense manipulative as per the larger objective of the study.

4. Conclusions

The study has attempted to derive the plausible design guidelines for designing spatial Number Sense manipulatives by involving children as the informants in the design process. Consistent feedback from the child-informants during the iterative action cycles had a direct impact on the design of subsequent prototypes and was thus pivotal in shaping the design process.

The study also illustrated the challenges of designing for such a young audience, and a possible approach for designing tangible learning products for them. From a designer, the study demanded patience, readiness to fail multiple times and swiftly adapt to the newly obtained insights, preparedness for a change in the course of plan based on the insights, and the ability to make quick and meaningful low fidelity prototypes.

Some of the behavioral characteristics of the young users unveiled during the study, such as their short attention span, affinity towards real-world contexts and stories, tendency to move around with the manipulatives in hand, an innate interest towards 'making' and 'play' etc., can act as crucial design considerations while designing of learning products for this age.

The study is a part of a larger ongoing study on the design of spatial-based Number Sense manipulatives for preschoolers. The hence derived insights and design criteria would hereafter assist the designer in designing the final set of spatial Number Sense manipulatives.

References

Pestalozzi, I-L (1803). ABC der Anschauung, oder Anschauungs-Lehre der Massverhaltnisse. Tubingen, Germany: J.G. Cotta

Montessori, M. (1912). The Montessori Method. New York Frederick Stokes Co.

Piaget, J. (1972). The Principles of Genetic Epistemology. New York Basic Books.

Resnick, M., Martin, F., Berg, R., Borovoy, R., Colella, V., Kramer, K., & Silverman, B. (1998, January). Digital manipulatives: new toys to think with. In *Proceedings of the SIGCHI conference on Human factors in computing systems*(pp. 281-287). ACM Press/Addison-Wesley Publishing Co..

Ackermann, E. (2001). Piaget's constructivism, Papert's constructionism: What's the difference. *Future of learning group publication*, *5*(3), 438.

Sarama, J., & Clements, D. H. (2016). Physical and Virtual Manipulatives: What Is "Concrete"?. In *International perspectives on teaching and learning mathematics with virtual manipulatives* (pp. 71-93). Springer, Cham.

Surender, E. (2019). Progressive Prototyping for the Design of Spatial-Number Sense Tools. In *DRS Learn X Design 2019, METU, Ankara.* Fifth International Conference for Design Education Researchers. Publishing under process

Boggan, M., Harper, S., & Whitmire, A. (2010). Using Manipulatives to Teach Elementary Mathematics. *Journal of Instructional Pedagogies*, *3*.

Druin, A. (2002). The role of children in the design of new technology. *Behaviour and information technology*, *21*(1), 1-25.

National Council of Teachers of Mathematics (Ed.). (2000). *Principles and standards for school mathematics* (Vol.1). Reston, USA: National Council of Teachers of Mathematics.

Berch, D.B. (2005). Making sense of number sense: Implications for children with mathematical disabilities. *Journal of learning disabilities*, *38*(4), 333-339.

Fosnot, C. T., & Dolk, M. (2001). *Young Mathematicians at Work: Constructing Number Sense, Addition, and Subtraction*. Heinemann, 88 Post Road West, PO Box 5007, Westport, CT 06881.

Boaler, J. (2015). Fluency without fear: Research evidence on the best ways to learn math facts. *Reflections*, *40*(2), 7-12.

Laski, E. V., Casey, B. M., Yu, Q., Dulaney, A., Heyman, M., & Dearing, E. (2013). Spatial skills as a predictor of first grade girls' use of higher level arithmetic strategies. *Learning and Individual Differences*, *23*, 123-130.

Casey, B. M., Lombardi, C. M., Pollock, A., Fineman, B., & Pezaris, E. (2017). Girls' spatial skills and arithmetic strategies in first grade as predictors of fifth-grade analytical math reasoning. *Journal of Cognition and Development*, *18*(5), 530-555.

Ontario Ministry of Education (2014). Paying attention to spatial reasoning K-12: Support document for Paying attention to mathematics education. Ontario: ServiceOntario.

Davis, B. (2015). Spatial Reasoning in the Early Years: Principles, Assertions, and Speculations. New York: Routledge

Kadosh, R.C., & Dowker, A. (Eds.). (2015). The Oxford Handbook of Numerical Cognition. Oxford Library of Psychology.

Sanders, E. B. N., & Stappers, P. J. (2014). Probes, toolkits and prototypes: three approaches to making in codesigning. CoDesign, 10(1), 5-14.

Kemmis, S., & McTaggart, R. (2005). Participatory action research: Communicative action and the public sphere. Sage Publications Ltd

The Metropolitan Museum of Art. (2004). *Islamic Art and Geometric Design* [Brochure]. New York: Yale University Press.

Nes, F. (2009). Young children's spatial structuring ability and emerging number sense (Ph.D. dissertation). Utrecht University, The Netherlands.

Brennan, K. (2014). Constructionism in the classroom: three experiments in disrupting technocentrism. Constructionism and Creativity, 1-8.